Shocks and Fermi-I Acceleration
Non-Relativistic Shocks

Stationary Frame

Shock Rest Frame

\[ p_1, \rho_1, T_1 \]

\[ v_s \]

\[ p_0, \rho_0, T_0 \]

\[ v_1 \]

\[ v_0 = -v_s \]

\[ p_1, \rho_1, T_1 \]

\[ p_0, \rho_0, T_0 \]
Particle Acceleration at Strong Shocks

General Idea:

Particles bouncing back and forth across shock front:

Stationary frame of ISM

\[ U = v_s \]

\[ \rho_2, \rho_2 \rightarrow \rho_1, \rho_1 \]

Shock rest frame

\[ v_2 = (1/4)v_1 \]

\[ v_1 = -U \]

\[ \rho_1 / \rho_2 = 1/4 \]

Rest frame of shocked material

\[ v_2' = (3/4)U \]

\[ v_1' = (3/4)U \]

At each pair of shock crossings, particles gain energy

\[ \langle \Delta E / E \rangle = (4/3) \frac{V}{c} = \frac{U}{c} \]

Write

\[ E = \beta E_0 = (1 + \frac{U}{c}) E_0 ; \quad \beta = 1 + \frac{U}{c} \]
Particle Acceleration at Strong Shocks (cont.)

Flux of particles crossing the shock front in either direction:

\[ F_{\text{cross}} = \frac{1}{4} N_c \]

Downstream, particles are swept away from the front at a rate:

\[ NV = \frac{1}{4} NU \]

Probability of particle to remain in the acceleration region:

\[ P = 1 - \left(\frac{\frac{1}{4} NU}{\frac{1}{4} N_c}\right) = 1 - \left(\frac{U}{c}\right) \]
Energy of a particle after $k$ crossings:

$$E = \beta^k E_0$$

Number of particles remaining:

$$N = P^k N_0$$

$$\Rightarrow \ln \left(\frac{N(>E)}{N_0}\right) / \ln \left(\frac{E}{E_0}\right) = \ln P / \ln \beta = -1$$

or

$$N(>E)/N_0 = (E/E_0)^{\ln P/\ln \beta}$$

$$\Rightarrow N(E)/N_0 = (E/E_0)^{-2}$$
More General Cases

Weak nonrelativistic shocks

\[ p > 2 \]

Relativistic parallel shocks:

\[ p = 2.2 – 2.3 \]

Relativistic oblique shocks:

Almost any spectral index possible
Diffusive Shock Acceleration

Monte Carlo Simulation Particle Trajectories
Diffusive Shock Acceleration

Particle retention in the shock layer is extremely sensitive to the magnetic field angle w.r.t. the shock normal in relativistic shocks.

Normal Incidence Frame (NIF)  de Hoffmann-Teller frame (HT)
Electron Spectra from Diffusive Shock Acceleration

\[ \lambda = \eta^* r_g = \text{Pitch-angle scattering mean free path} \]

**Moderately sub-luminal**

\( \beta_{1HT} = \beta_{1x}/\cos\Theta_{Bf1} < 1 \)

**Marginally sub-luminal**

\( \beta_{1HT} = \beta_{1x}/\cos\Theta_{Bf1} \sim 1 \)

(Summerlin & Baring 2012)
Asymptotic Particle Spectral Index

\[ n(\gamma) \sim \gamma^{-\sigma} \]

(Summerlin & Baring 2012)
Effects of Cooling and Escape

Evolution of particle spectra is governed by the Continuity Equation:

$$\frac{\partial n_e(\gamma,t)}{\partial t} = -\frac{\partial}{\partial \gamma} (\gamma n_e) + Q_e(\gamma,t) - \frac{n_e(\gamma,t)}{t_{\text{esc},e}}$$

- Radiative and adiabatic losses
- Particle injection (acceleration on very short time scales)
- Escape
Effects of Cooling and Escape (cont.)

Assume rapid particle acceleration:

\[ Q(\gamma, t) = Q_0 \gamma^q \quad \gamma_1 < \gamma < \gamma_2 \]

Fast Cooling:

\[ t_{\text{cool}} \ll t_{\text{dyn}}, t_{\text{esc}} \] for all particles

Particle spectrum:

- Synchrotron or Compton spectrum:
Effects of Cooling and Escape (cont.)

Assume rapid particle acceleration:

$$Q(\gamma, t) = Q_0 \gamma^q \quad \gamma_1 < \gamma < \gamma_2$$

Slow Cooling:

$$t_{cool} << t_{dyn}, t_{esc}$$ only for particles with $$\gamma > \gamma_b$$

Particle spectrum:

- Synchrotron or Compton spectrum:

$$F(v) = v^{-(q-1)/2}$$